Critical Slowing Down along the Dynamic Phase Boundary in Ising meanfield dynamics

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Abstract: We studied the dynamical phase transition in kinetic Ising ferromagnets driven by oscillating magnetic field in meanfield approximation. The meanfield differential equation was solved by sixth order Runge-Kutta-Felberg method. The time averaged magnetisation plays the role of the dynamic order parameter. We studied the relaxation behaviour of the dynamic order parameter close to the transition temperature, which depends on the amplitude of the applied magnetic field. We observed the critical slowing down along the dynamic phase boundary. We proposed a power law divergence of the relaxation time and estimated the exponent. We also found its dependence on the field amplitude and compared the result with the exact value in limiting case.

Keywords: Ising model, Meanfield theory, Dynamic transition, Relaxation time, Critical slowing down

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I. Introduction:

The kinetic Ising model driven by an oscillating magnetic field yields various nonequilibrium response [1]. One interesting nonequilibrium response is the dynamic phase transition. This dynamic phase transition is widely studied in model ferromagnetic system in the presence of oscillating magnetic field [1]. Tome and Oliveira [2] first observed a prototype of nonequlibrium dynamic transition in the numerical solution of meanfield equation of motion for the classical Ising ferromagnet in the presecce of a magnetic field varying sinusoidally in time. The time averaged (over the complete cycle of the oscillating magnetic field) magnetisation plays the role of the dynamic order parameter. They [2] found that this dynamic ordering depends on the amplitude of the oscillating magnetic field and the temperature of the system. Systems get dynamically ordered for small values of the temperature and the amplitude of the field. They [2] have drawn a phase boundary (separating the ordered and disordered phase) in the temperature field amplitude plane. More interestingly, they have also reported [2] a tricritical point on the phase boundary, which separates the nature (continuous/discontinuous) of the dynamic transition across the phase boundary. This tricritical point was found just by checking the nature of the transition at all points across the phase boundary. The point where the nature of transition changes was marked as the tricritical point. No other significance of this tricritical point was reported. The frequency dependence of this phase boundary was not reported earlier for the dynamic transition in Ising meanfield dynamics.

We have studied [3] numerically the frequency dependence of the dynamic phase boundary in Ising meanfield dynamics. We studied the tricritical behaviour and found a method of finding the position the tricritical point on the dynamic phase boundary. The frequency dependence of the position of the tricritical point was studied here. We also studied the static (zero frequency) limit of dynamic phase boundary.

The divergence of the time scale, near the dynamic transition temperature was studied [5] by Monte Carlo simulation and by solving meanfield dynamical equation in driven Ising ferromagnet. The critical slowing down was observed in both cases. In meanfield case, the exponent was also calculated [5] in the limit of vanishingly small field amplitude. However, it would be interesting to know how this exponent depends on the field amplitude along the dynamic phase boundary. In this paper, we have studied this and also checked the earlier result of the value of this exponent in the limit of

vanishingly small field amplitude.

The paper is organised as follows: In the next section the model and the method of numerical solution is discussed. Section III contains the numerical results and the paper end with summary of the work in section IV.

II. Model and numerical solution:

The time (t) variation of average magnetisation m of Ising ferromagnet in the presence of a time varying field, in meanfield approximation, is given as [2]

$$\tau \frac{dm}{dt} = -m + \tanh(\frac{m + h(t)}{T}),\tag{1}$$

where, $h(t)$ is the externally applied sinusoidally oscillating magnetic field $(h(t) = h_0 \sin(\omega t))$ and T is the temperature measured in units of the Boltzmann constant (K_B). This equation describes the nonequilibrium behaviour of instantaneous value of magnetisation $m(t)$ of Ising ferromagnet in meanfield approximation. Here, τ stands for the microscopic relaxation time for the spin flip [2].

In this context, the Hamiltonian describing the Ising ferromagnet may be written as follows:

$$H = -J\Sigma_{<ij>}S_iS_j - h(t)\Sigma_i S_i \tag{2}$$

where the symbols express their usual meaning [5].

We have solved this equation by sixth order Runge-Kutta-Felberg (RKF) [4] method to get the instantaneous value of magnetisation $m(t)$ at any finite temperature T, h_0 and $\omega(= 2\pi f)$. This method of solving ordinary differential equation $\frac{dm}{dt} = F(t, m(t))$, is described briefly as:

$m(t + dt) = m(t) + \left(\frac{16k_1}{135} + \frac{6656k_3}{12825} + \frac{28561k_4}{56430} - \frac{9k_5}{50} + \frac{2k_6}{55} \right)$
where
$k_1 = dt \cdot F(t, m(t))$
$k_2 = dt \cdot F(t + \frac{dt}{4}, m + \frac{k_1}{4})$
$k_3 = dt \cdot F(t + \frac{3dt}{8}, m + \frac{3k_1}{32} + \frac{9k_2}{32})$
$k_4 = dt \cdot F(t + \frac{12dt}{13}, m + \frac{1932k_1}{2197} - \frac{7200k_2}{2197} + \frac{7296k_3}{2197})$
$k_5 = dt \cdot F(t + dt, m + \frac{439k_1}{216} - 8k_2 + \frac{3680k_3}{513} - \frac{845k_4}{4104})$
$k_6 = dt \cdot F(t + \frac{dt}{2}, m - \frac{8k_1}{27} + 2k_2 - \frac{3544k_3}{2565} + \frac{1859k_4}{4104} - \frac{11k_5}{40})$..............................(2)

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The time interval dt was measured in units of τ (the time taken to flip a single spin). Actually, we have used $dt = 0.01$ (setting $\tau=1.0$). The local error involved in the sixth order RKF method is of the order of $(dt)^6 (= 10^{-12})$. We started with initial condition $m(t = 0) = 1.0$. In the present case, we kept the frequency $f = 0.1$, fixed throughout the study.

III. Results:

The time averaged magnetisation over a full cycle of the oscillating magnetic field acts as the role of dynamic order parameter $Q(= \frac{\omega}{2\pi} \oint m(t)dt)$. For steady state calculations [3], this was considered after discarding the values of Q for few initial (transient [5]) cycles of the oscillating field. However, in this paper, we are interested in the transient behaviour of the dynamic order parameter Q. This Q was calculated for every cycle (of the oscillating magnetic field) starting from the first cycle. The dynamic order parameter Q was studied as a function of the number of cycles n. This shows a relaxation behaviour of Q (Fig-1). Figure-1 shows the variations of $log(Q)$ with respect to n for different temperatures. Here, the frequency $f = 0.1$ and the field amplitude $h_0 = 0.2$ remains same. Since, we remain in the disordered phase the dynamic order parameter Q will vanish as n increases. Here, as the temperature T approaches the dynamic transition temperature the relaxation becomes slower. This is a clear indication of *critical slowing down*. Moreover, since the semilog plot of Q is linear, the relaxation is *exponential* and one may expect the behaviour like $Q \sim exp(-n/\Gamma)$, where Γ defines the relaxation time. We calculated the relaxation time Γ from the least square fit of the $log(Q) - n$ data (discarding the initial nonlinear part for small n).

For a fixed value of the field amplitude h_0, the relaxation time Γ was studied as a function of temperature T. It is observed that Γ diverges as the temperature approaches the dynamic transition temperature $T_d(h_0)$. This is demonstrated in Figure-2, for three different values of field amplitudes (h_0). Here, we observed the critical slowing down along the dynamic phase boundary. We assumed the scaling law (if valid still in nonequilibrium case), $\Gamma \sim (T - T_d(h_0))^{-z}$ and estimated the exponent z as well as $T_d(h_0)$ numerically, for different values of h_0. If our scaling assumption is correct, the variation of $(\frac{1}{\Gamma})^{(\frac{1}{z})}$ with T will be a straight line. The straight line cuts the temperature axis at $T = T_d(h_0)$. We employed the numerical method to estimate z and $T_d(h_0)$. The method is as follows: we first use a trial value

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of z and calculated $(\frac{1}{\Gamma})^{(\frac{1}{z})}$ for various values of T. The data were fitted to a straight line of the form $y = mx + c$. The error $e = \sum_i (y_i - mx_i - c)^2$ was calculated for various values of z. The accepted value of z was found for which the error e becomes minimum. This is depicted in Figure-3a. Here, $h_0 = 0.2$ and e becomes minimum for $z \cong 1$. The best (least square) fit straight line was shown in Figure-3b. By extrapolating the line one gets $T_d(h_0 = 0.2) = 0.9239$. Similar methods were employed to get the values of z for other different values of $h_0 = 0.3$ and $h_0 = 0.4$. For $h_0 = 0.3$, we estimated $z = 0.993$ and $T_d(h_0 = 0.3) = 0.8115$. The corresponding best fit straight line plot is shown in Figure-4. Figure-5 shows the best fit straight line plot of $(\frac{1}{\Gamma})^{(\frac{1}{z})}$ versus T, which estimates $z = 0.981$ and $T_d(h_0 = 0.4) = 0.6318$.

IV. Summary:

In this paper, we have reported our numerical results of the study of the relaxation behaviour of the dynamic order parameter in the dynamic phase transition in Ising meanfield dynamics. The relaxation was observed to be exponential in nature and the relaxation time was found to vary as $(T - T_d(h_0))^{-z}$. We estimated z (as well as $T_d(h_0)$) numerically and found it to vary with h_0. We checked the value of z in the limit of $h_0 \to 0$ obtained analytically [5].

Our present study has the two importances. Firstly, it proves that the critical slowing down of the dynamic order parameter along the dynamic phase boundary has power law scaling behaviour (at least in the present case), usually observed in equilibrium critical phenomena. Secondly, the exponent z is a monotonically decreasing function of h_0 and approaches the exact value $(z = 1)$ [5] in the limit $h_0 \to 0$.

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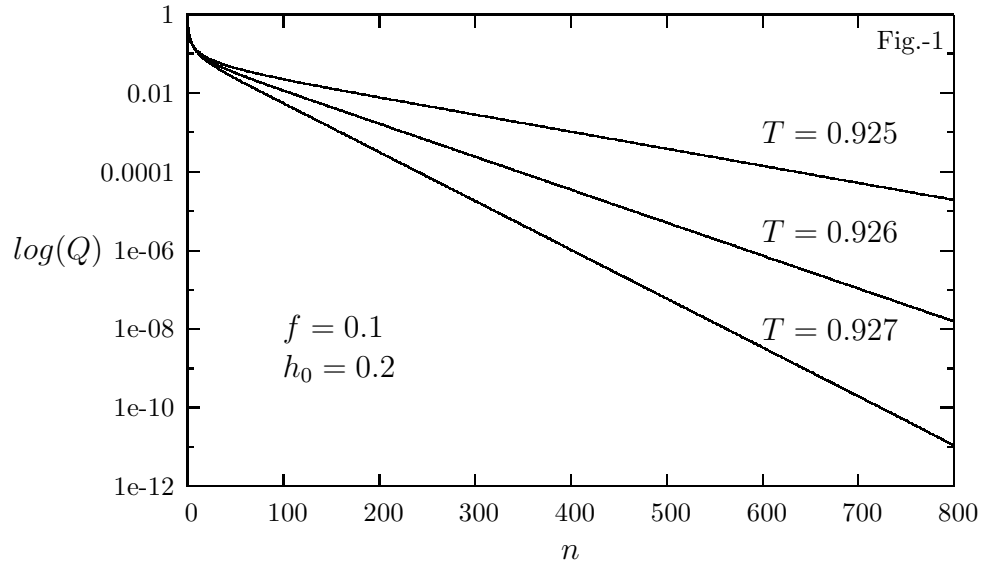
Fig.-1. The Exponential relaxation of the dynamic order parameter Q at different temperatures. Relaxation is faster at higher temperature.
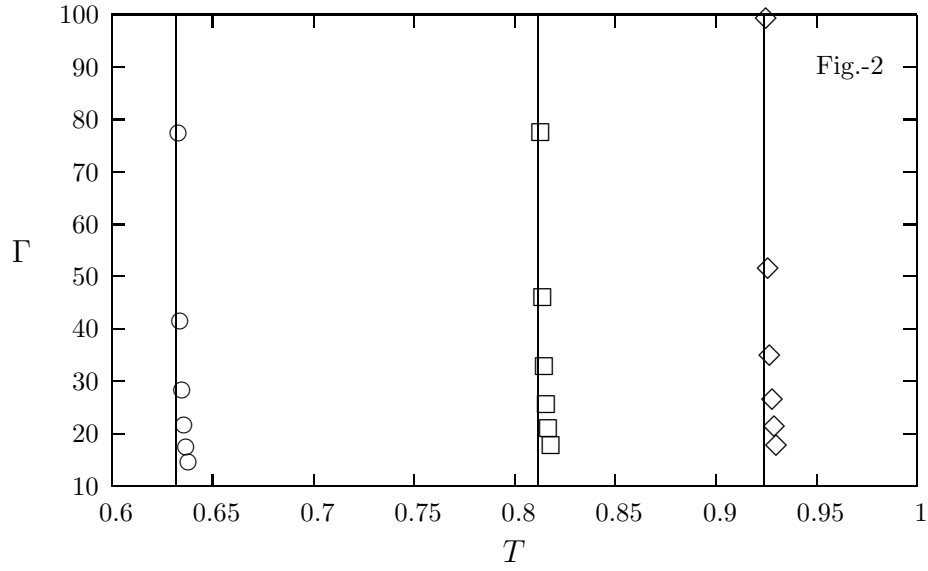
Fig.-2. The critical slowing down. The relaxation time diverges near the dynamic transition points $T_d(h_0)$. Different symbols represents different values of h_0. $h_0 = 0.4(O)$, $h_0 = 0.3(\square)$ and $h_0 = 0.2(\diamond)$.
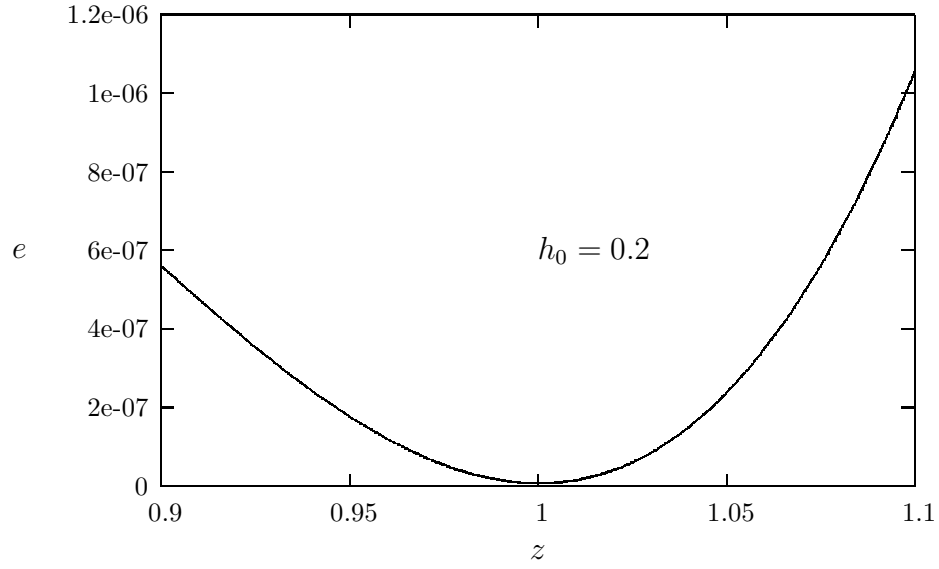
Fig.3a. The variation of the error (e) of straight line fit of $(\frac{1}{\Gamma})^{(\frac{1}{z})}$ versus T for various values of z. Here, $h_0 = 0.2$. Note that the error is minimum for $z \cong 1.0$.
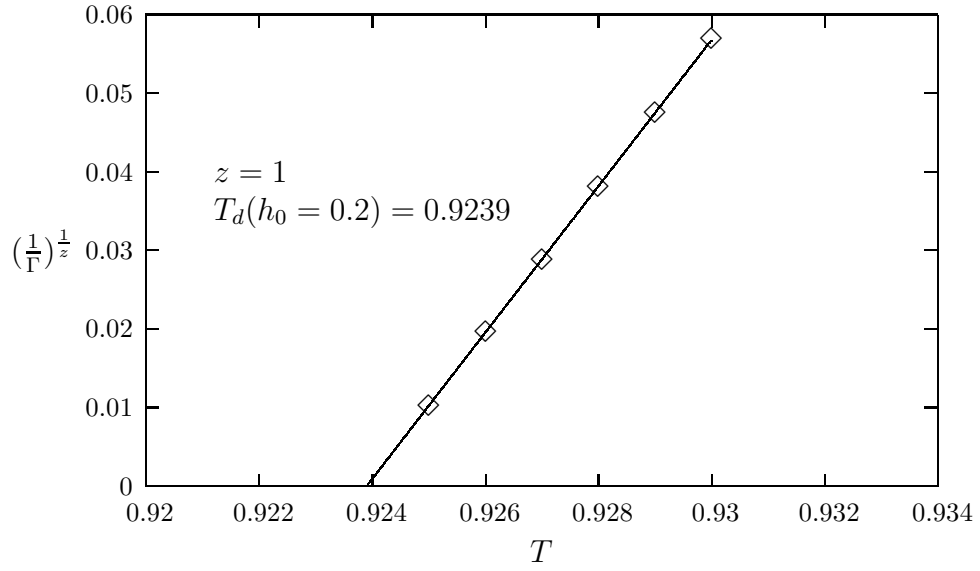
Fig.3b. The best (least error) fit straight line to estimate z and $T_d(h_0)$ for $h_0 = 0.2$.
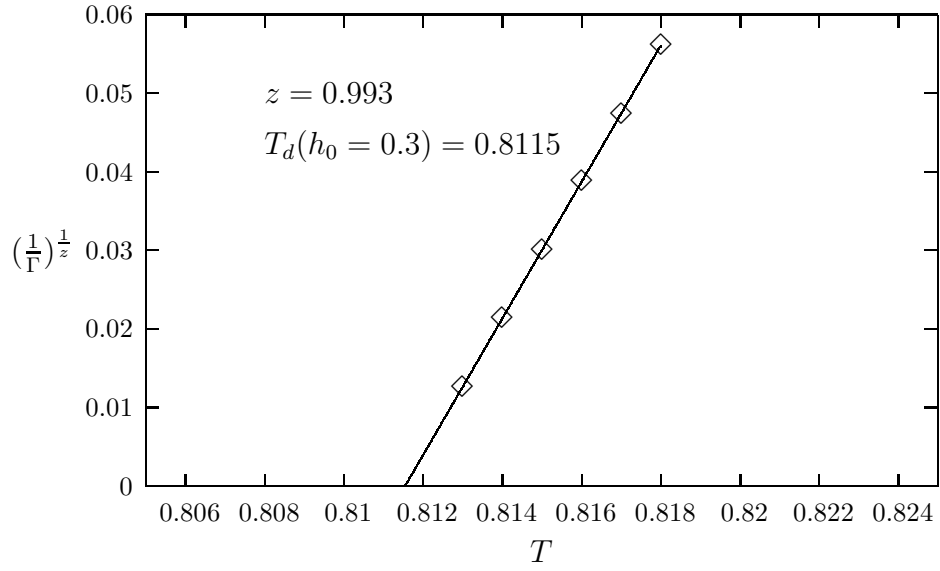
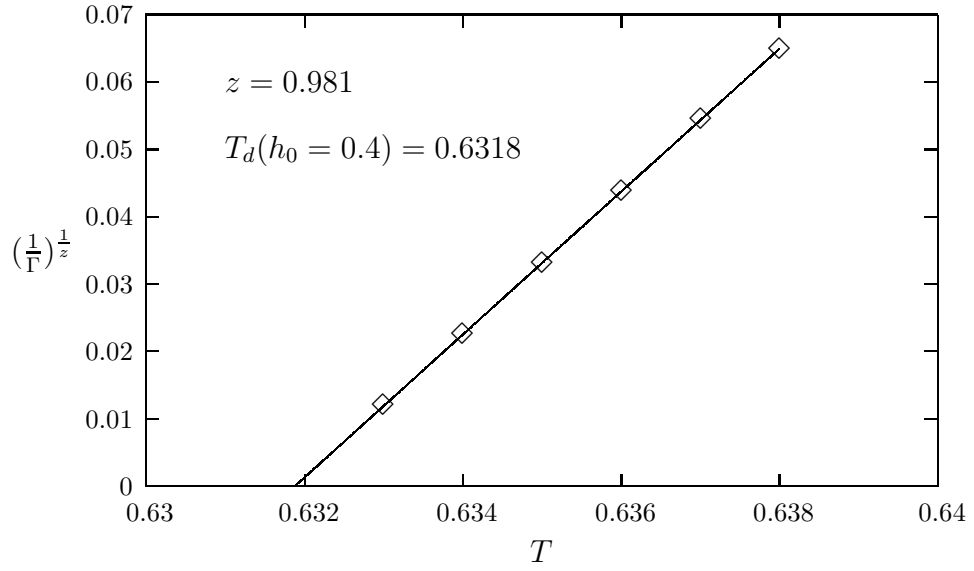Fig.4. The best (least error) fit straight line to estimate z and $T_d(h_0)$ for $h_0 = 0.3$.

Fig.5. The best (least error) fit straight line to estimate z and $T_d(h_0)$ for $h_0 = 0.4$.